

High-Temperature Creep Under a Nonuniform Temperature Distribution

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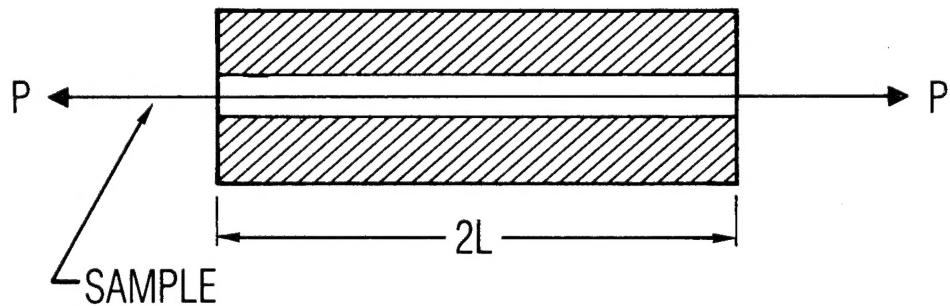
The dimensional stability of a material often plays an important role in the use of that material at high temperature, particularly in structural applications. The determination of creep behavior at high temperature is frequently complicated by a nonuniform temperature distribution. In order to approximate the ideal situation of a uniform temperature distribution, various schemes are used, such as reducing the cross section within the hot zone to restrict the highest stress to a suitably small region. However, in many practical situations, as well as in experiments dealing with the measurement of creep in composites (Ref. 1), a material is inevitably subjected to a nonuniform temperature distribution. The total creep elongation is then a sum of contributions from material elements with a distribution of temperatures.

In this report, we analyze an example of creep in a material under a nonuniform temperature distribution, and we use simple but realistic assumptions for that distribution and for the material creep properties as a function of temperature. This analysis was motivated by measurements of creep at high temperatures in unidirectional, fibrous carbon-carbon composites (Refs. 2 and 3).

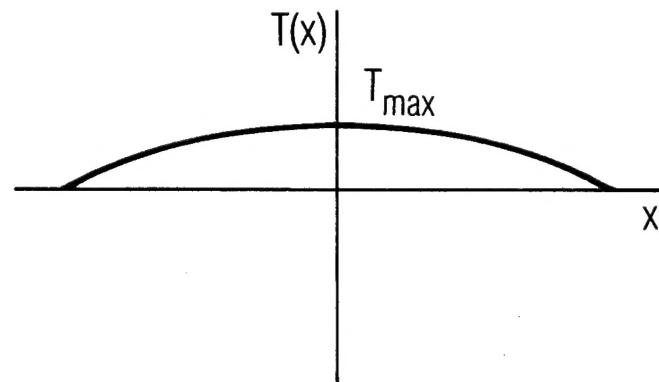
A sample of uniform cross section was used because of ease of fabrication. The sample was placed in a furnace with a central peak temperature. A tensile load was applied to the sample and creep elongation was measured. However, creep rates versus temperature were not obtainable directly from the creep data, due to the temperature distribution across the sample. The following analysis allowed us to predict both creep rates at a uniform temperature and elongation rates under other, nonuniform temperature distributions.

The sample in our model is long and thin (depicted schematically in Fig. 1), having length $2L$ and uniform cross-sectional area A in the high-temperature region. We assume that all strains are small, $\epsilon \ll 1$, and that the sample is under tension, P , or uniaxial tensile stress, $\sigma = P/A$. Creep rate, $\dot{\epsilon}(x)$, and the temperature distribution, $T(x)$, are functions of position along the sample axis, x .

FURNACE



TEMPERATURE PROFILE



CREEP RATE PROFILE

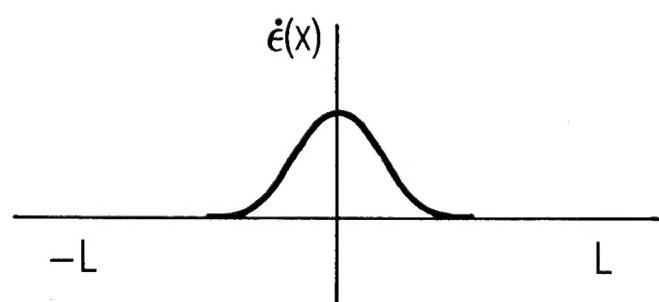


Fig. 1. Schematic of idealized experiment.

The average creep rate over the sample length is given by

$$\langle \dot{\epsilon} \rangle = \frac{1}{2L} \int_{-L}^{L} \dot{\epsilon}(x) dx \quad (1)$$

We assume the material has a simple Dorn-Weertman (Ref. 4) temperature-dependent creep behavior:

$$\dot{\epsilon} = \dot{\epsilon}_0(\sigma) \exp(-E/kT) \quad (2)$$

where $\dot{\epsilon}_0(\sigma)$ is a rate coefficient, σ is stress, E is activation energy, k is Boltzmann's constant, and T is absolute temperature. A simple expression for the average creep rate under a nonuniform temperature distribution is obtained by inserting Eq. (2) into Eq. (1):

$$\langle \dot{\epsilon} \rangle = \frac{1}{2L} \int_{-L}^{L} \dot{\epsilon}_0(\sigma) \exp[-E/kT(x)] dx \quad (3)$$

We also assume the parabolic temperature distribution (Refs. 3 and 5):

$$T(x) = T_{\max} \left[1 - \left(\frac{x}{L} \right)^2 \right] \quad (4)$$

with a peak temperature, T_{\max} . For convenience, the temperature at the ends of the sample is taken to be absolute zero, which is a reasonable approximation because the contribution to the integral from the regions of the sample near the ends is small. We define $\alpha = E/kT_{\max}$ to be a dimensionless activation energy, and eliminate x in favor of q , a dimensionless variable, in Eqs. (3) and (4) through the substitution $x/L = [1 + (\alpha/q)]^{-1/2}$. Using the fact that $T(x)$ is an even function of x , we then obtain

$$\frac{\langle \dot{\epsilon} \rangle}{\dot{\epsilon}_0(\sigma)} = \frac{1}{2} \alpha^{-1/2} e^{-\alpha} \int_0^{\infty} e^{-q} q^{-1/2} (1 + \frac{q}{\alpha})^{-3/2} dq \quad (5)$$

This integral can be easily evaluated by noting the results of experiments on high-temperature creep in carbon-carbon composites (Ref. 2). Activation energies of the order of 100 kcal/mole were obtained at temperatures of around 2500 K, thus giving values of α of ~20. Consequently, we can expand the binomial in the integrand in powers of q/α and integrate term by term. Because the integral is over all $q > 0$, we integrate the series outside its radius of convergence, which results in an expression for the integral that is an asymptotic (rather than convergent) series (Ref. 6) in powers of $1/\alpha$:

$$\begin{aligned} \frac{\langle \dot{\epsilon} \rangle}{\dot{\epsilon}_0(\sigma)} &= \frac{1}{2} \left(\frac{\pi}{\alpha}\right)^{1/2} e^{-\alpha} \sum_{k=0}^{\infty} (-1)^k \left(\frac{1}{\alpha}\right)^k \frac{(2k+1)!}{(k!)^3} \frac{(2k)!}{2^{4k}} \\ &= \frac{1}{2} \left(\frac{\pi}{\alpha}\right)^{1/2} e^{-\alpha} \left[1 - \frac{3}{4} \left(\frac{1}{\alpha}\right) + \frac{45}{32} \left(\frac{1}{\alpha}\right)^2 - \frac{525}{128} \left(\frac{1}{\alpha}\right)^3 + \dots \right] \quad (6) \end{aligned}$$

An alternative approach to evaluating the integral in Eq. (5) is the method of steepest descents (Ref. 6), an approach that gives the first term in the series in Eq. (6).

For experimentally relevant values of α , the series in Eq. (6) gives a good approximation to the integral, to within a few percent, with only the first two terms.* For a value of $\alpha \sim 20$, it can be seen from Eq. (6) that the first-order term in $1/\alpha$ gives a correction of less than 4%. This suggests in fact that, to a reasonable approximation,

$$\frac{\langle \dot{\epsilon} \rangle}{\dot{\epsilon}_0(\sigma)} = \frac{1}{2} \left(\frac{\pi}{\alpha}\right)^{1/2} e^{-\alpha} \quad (7)$$

Using Eq. (7) to analyze the elongation rates at different peak temperatures, we can obtain the activation energy, E , and pre-exponential $\dot{\epsilon}_0(\sigma)$.

*In general, best precision is obtained with the series by locating the minimum term and truncating the series one term before it, see Ref. 6.

The result in Eq. (7) can be viewed in terms of an "effective gauge length," L_{eff} , defined by

$$\langle \dot{\epsilon} \rangle L = \dot{\epsilon}_0(\sigma) e^{-\alpha} L_{\text{eff}} = \dot{\epsilon} L_{\text{eff}} \quad (8)$$

where $\dot{\epsilon}$ is the strain rate for a sample under a uniform temperature equal to T_{max} , given in Eq. (2).

For the temperature distribution in Eq. (4),

$$L_{\text{eff}} = \frac{1}{2} \left(\frac{\pi}{\alpha} \right)^{1/2} L \quad (9)$$

Thus, in a typical experimental example (Refs. 2 and 3), for $\alpha = 20$ and a sample length, $2L$, of 91.4 cm (36 in.),

$$L_{\text{eff}} = \frac{1}{2} \left(\frac{\pi}{20} \right)^{1/2} (45.7 \text{ cm}) = 8.9 \text{ cm (3.5 in.)}$$

Equation (2) can be used with L_{eff} to obtain $\dot{\epsilon}(2L_{\text{eff}})$, which is the elongation rate, and which gives total elongation when multiplied by the time interval.

A problem mathematically similar to that of calculating creep under a nonuniform temperature distribution is determining the amount of diffusion or degree of heat treatment when the sample is held at a uniform temperature, but the temperature varies as a function of time, $T(t)$. For example, following Shewmon (Ref. 7), the mean-squared diffusion distance is expressed as

$$\langle x^2 \rangle = 2 D(T)t \quad (10)$$

where the diffusion constant $D(T)$ is typically of the form

$$D = D_0 \exp(-E/kT)$$

If temperature varies with time as $T(t)$, then the total mean-squared diffusion distance over a time t' is

$$\langle x^2 \rangle = 2 \int_0^{t'} D[T(t)] dt \quad (11)$$

In conclusion, for thermally activated creep we have shown that an asymptotic expansion for the average elongation rate can be used to determine the creep parameters $\alpha = E/kT_{\max}$ and the pre-exponential factor $\epsilon_0(\sigma)$. We have done this for a parabolic temperature distribution, but the same approach can be used for other temperature distributions. Having determined the parameters, we can use the expression in Eq. (3) to predict the elongation rate for any given temperature distribution. In addition, we have indicated that analogous problems, such as heat treatment, are of similar mathematical form.

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